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Research Article

Coupling of Peridynamics and Timoshenko Beam Theory for the Stress Analysis of Laminated Composite Materials

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Abstract

This study investigates the bending behavior of laminated composite beams by using Peridynamic Least Squares Minimization (PDLSM) approach and Timoshenko Beam Theory (TBT). The PDLSM converts any arbitrary order derivatives of a function in their nonlocal forms with high accuracy. Therefore, it is highly suitable for the solution of TBT equilibrium equations. The influence of span-to-thickness ratios, loading and boundary conditions, as well as laminations on the stress and deformation fields of laminated composite beams, were investigated in detail. Also, the PDLSM was used for the transverse shear stress calculations from the stress equilibrium equations. It was demonstrated that the PD-TBT successfully predicted the deformation and stress fields in the laminated beams.

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Tabakalı Kompozit Malzemelerin Gerilme Analizleri için Peridinamik ve Timoşenko Kiriş Teorisinin Birleştirilmesi

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Öz

Bu çalışmada, Peridinamik En Küçük Kareler Minimizasyonu (PEKKM) yaklaşımı ve Timoşenko Kiriş Teorisi (TKT) kullanılarak tabakalı kompozit yapıya sahip kirişlerin eğilme davranışı incelenmiştir. PEKKM, bir fonksiyonun herhangi mertebeden türevini yerel olmayan integral formlarında doğru bir şekilde hesaplamaktadır. Bu sebepted dolayı, PEKKM, TKT denge denklemlerinin çözümü için oldukça uygun bir yaklaşımdır. Tabakalı yapıya sahip kirişlerin en-boy oranlarının, yükleme ve sınır koşullarının yanı sıra fiber açılarının bu yapılardaki gerilme ve şekil değiştirme alanlarına etkisi ayrıntılı olarak incelenmiştir. Ayrıca, PEKKM yaklaşımı, kayma gerilme değerlerinin, gerilme denge denklemlerini kullanılarak elde edilmesi için kullanılmıştır. Mevcut yaklaşımın tabakalı yapıya sahip kompozit kirişlerdeki şekil değiştirme ve gerilme dağılımlarını başarıyla tahmin ettiği gösterilmiştir.

1. INTRODUCTION

Composite materials possess many superior properties because of their weight, strength, and durability; thus, the application of such structures can be found in different engineering fields such as aerospace and military to name a few. The presence of multiple materials in these structures, the damage may take place as matrix breaking, fiber breaking, and delamination. In order to gain reliable composite models, their behaviors under loads must be well understood. The experimental investigations may be expensive and time-consuming [1]. Therefore, many research on the analyses of composite materials have focused on accurate and robust numerical tools.

Finite Element Method (FEM) is mostly employed to examine the deformation and stress fields of multi-layered composite structures since it provides a way to model complex domains. Generally, laminated composite structures are made of a combination of different types of materials having various layer thicknesses. Modeling of laminated composites may be computationally cumbersome owing to the requirement of a high fidelity mesh for the accuracy of the solution. The equivalent single layer (ESL) theories have been used to obtain efficient and accurate analyses of the composite beam, plate, and shell structures [2,3]. These theories consider constant kinematic variables (unknown) on the mid-plane regardless of the number of material layers along the cross-section of laminates.

One of the most well-known and the simplest ESL theories is Euler-Bernoulli Beam Theory (EBT) namely classical beam theory (CBT). It is suitable for modeling thin structures since it disregards the rotational and shear deformations. Timoshenko Beam Theory (TBT) is the most popular ESL theory. Unlike the EBT, TBT paves the way for the analysis of moderately thick beams due to the presence of transverse shear deformations in the theory. The equilibrium equations of the ESL theories are expressed in terms of spatial derivatives. Hence, it fails to approximate the solutions if the domain involves discontinuities such as cracks and sharp gradients in different materials. In order to alleviate the abovementioned problems more conveniently, mesh-free methods such as radial basis functions (RBF), element free galerkin methods (EFG), reproducing kernel particle method (RKPM), smoothed particle hydrodynamics (SPH), and Peridynamics (PD) have been devised [4]. Meshless methods remove the reliance on the mesh topology and consider the interactions of material points within a finite zone. PD theory was introduced by Silling [5] in 2000. It is a reformulation of the equilibrium equations in the classical continuum mechanics (CCM) and includes integral expressions rather than spatial derivatives of displacement components [6]. Therefore, the equilibrium equations in the PD theory remains valid even if the solution domain consists of singularities such as cracks and interfaces.

In recent years, interest in meshless methods has grown rapidly. Garg et al. [7] presented a detailed review on the meshless methods. Nguyen et al. [4] and Liew et al. [8] carried out a literature survey to overview on the recent advances in the meshless methods and their applications for modeling the laminated composite structures. Ferreira [9] used RBF and TBT for monitoring the stress fields in laminated composite beams under transverse loads. Gherlone et al. [10] investigated the bending behavior of the multilayered composite plates by using the first-order shear deformation theory (FSDT) which is the extension of TBT for plates and RBF for various boundary and loading conditions. Karamanli [11] employed SPH and TBT to examine the flexural behavior of simply supported and clamped laminated beams. Dorduncu et al. [12] examined the flexure behavior of laminated composite plates by solving the equilibrium equations of the FSDT within the light of PD theory.

O'Grady and Foster [13,14] introduced a PD formulation for the classical beam (EBT) and plate, namely Kirchhoff-Love theories. Vu et al. [15] investigated the dynamic and static behaviors of functionally graded plates by combining the FSDT and moving Kriging (MK) method. Diyaroglu et al. [16] used PD form of TBT and FSDT formulations for multi-layered beams and plates, respectively.

Meshless methods possess many advantages comparing the local methods, yet they may fail to produce accurate predictions due to the inappropriate shape parameters, nonsymmetric kernel functions, and the implementation of the boundary conditions. Madenci et al. [17,18] introduced a novel Peridynamic Least Squares Minimization (PDLSM) approach to remedy the shortcomings of the existing methods for the calculation of derivatives. In the PDLSM approach, the local derivates are written in their integral forms. It enables the determination of derivatives in the solution domains with discontinuities. It enables the use of both symmetric and non-symmetric domains of interaction at a PD point in the domain; thus, it removes the requirement of fictitious PD points along the boundaries. The PDLSM approach was successfully implemented in the area of PD theory. Madenci et al. [19,20] developed a weak form of PD and coupled FEM-PD theories within the framework of PDLSM. The robustness of the PDLSM was demonstrated for the stress analysis of laminated composite beams [21-23] and plates [24] by using refined zigzag theory (RZT).

This study aims to develop a new solution scheme for the solution of the TBT equilibrium equations within the framework of the PDLSM approach for the first time. The accuracy and robustness of the present approach called PD-TBT are constructed against the reference solutions [11,25,26] for various boundary and loading conditions as well as laminations in the beam.

2. PERIDYNAMIC LEAST SQUARES MINIMIZATION

2.1. Concept of Peridynamics

Peridynamic (PD) theory is a mesh-free nonlocal theory. Unlike the local theories such as FDM and FEM, in the PD theory, interactions between PD points exist in a finite region (horizon). Figure 1 shows a solution domain of D for the PD analyses. PD points have their own family (the domain of interaction) namely horizon ($H_{\mathbf{x}}$). The PD point x interacts with many PD points, x' in its domain of interaction. In the undeformed state, the material points x and x' has a relative distance of $\xi = x' - x$. The PD point has an entity (length) of ℓ_x and the horizon size of $\delta_x = m\ell_x$. m specifies the size of the horizon and requires convergences studies for an appropriate horizon size.

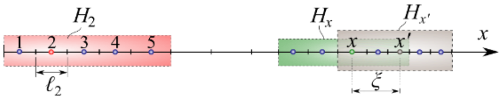

Figure 1. A PD solution domain and PD interactions between points in their families

2.2. Peridynamic Least Squares Minimization (PDLSM)

PDLSM has been recently developed by Madenci et al. [17,18] for the approximation of any arbitrary derivatives of a function $f(x)$ in an integral PD form.

Employing the concept of PD interactions and Least Squares Minimization for the determination of any arbitrary derivation of a function $f(x)$ and expanding the Taylor's series expansion (TSE) up to N-th order leads to

$$f(x') = \sum_{n=0}^{N} \frac{1}{n!} \xi^n \frac{d^n f(x)}{dx^n} + R(N,x) \qquad (1)$$

where the small reminder term $R(N,x)$ can be neglected. The error, E_x, at the PD point x can be expressed in the nonlocal form as

$$E_x = \int_{H_x} w(\xi) \left(f(x') - \sum_{n=0}^{N} \frac{1}{n!} \xi^n \frac{d^n f(x)}{dx^n} \right)^2 d\xi \qquad (2)$$

where $w(\xi)$ represents the nondimensional weight function. The minimization of the error can be achieved if the first variation of the error is $\delta E_x = 0$ and can be written as

$$\delta\left(\frac{d^k f(x)}{dx^k}\right) E_x = 0 \qquad (3)$$

where δ is a variational operator and k varies from 0 to N. Considering any arbitrary $\delta\left(d^k f(x)/dx^k\right)$, Eq.(3) results in a system equations as

$$A_{kn}\left(\frac{1}{n!}\frac{d^n f(x)}{dx^n}\right) = M_k \qquad (4)$$

where A_{kn} and M_k can be written as

$$A_{kn} = \int_{H_x} w(\xi) \xi^{k+n} d\xi \qquad (5a)$$

and

$$M_k = \int_{H_x} \xi^k w(\xi) f(x') d\xi \qquad (5b)$$

The PD form of local derivatives for arbitrary order can be expressed as

$$\frac{d^n f(x)}{dx^n} = n! \int_{H_x} \left(\sum_{k=0}^{N} A_{nk}^{-1} \xi^k \right) w(\xi) f(x') d\xi \qquad (6)$$

3. TIMOSHENKO BEAM THEORY

Figure 2 shows a beam with a length of *L*, a width of *b*, a thickness of *2h*, and *N* material layers under a transversely distributed load *p*(*x*). The thickness of each material layer can be arbitrarily specified as $2h^{(k)}$. The Timoshenko Beam Theory possesses three kinematic variables through the thickness of the beam within a region defined as $x \in [0, L]$ (axial) and $z \in [-h, h]$ (thickness). The displacement field in the k^{th} layer can be expressed in the form of

$$u_x^{(k)}(x,z) = u(x) + z\theta(x) \qquad (7a)$$

$$u_z^{(k)}(x,z) = w(x) \qquad (7b)$$

where $u_x^{(k)} = u_x^{(k)}(x,z)$ and $u_z^{(k)} = u_z^{(k)}(x,z)$ are referred to the axial and transverse displacement components, respectively. $u = u(x)$ denotes the mid-surface axial displacement, $\theta = \theta(x)$ is the average slope, and $w = w(x)$ represents the deflection.

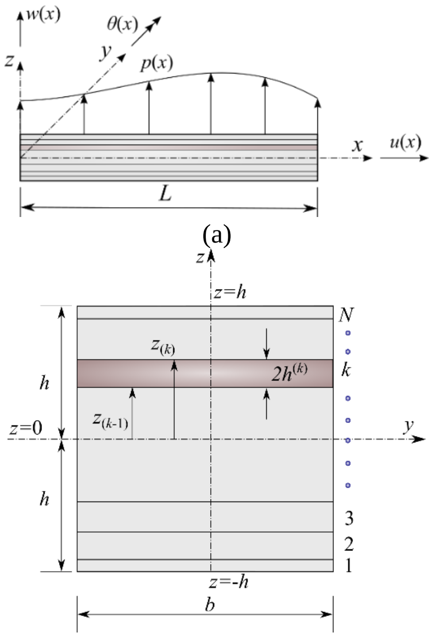

Figure 2. (a) Kinematics of TBT and (b) the cross-section of a beam

The linear axial, $\varepsilon_{xx}^{(k)}$, and transverse shear strain , $\gamma_{xz}^{(k)}$, components of the k^{th} layer can be written in terms of the displacement components of the TBT (Eq. 7) as

$$\begin{Bmatrix} \varepsilon_{xx}^{(k)} \\ \gamma_{xz}^{(k)} \end{Bmatrix} = \begin{bmatrix} 1 & z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_{,x} \\ \theta_{,x} \\ w_{,x} + \theta \end{Bmatrix} \qquad (8)$$

in which the comma indicates the differentiation with respect to *x*. The axial, $\sigma_{xx}^{(k)}$, and transverse shear stress, $\sigma_{xz}^{(k)}$, components of the k^{th} material layer are obtained by employing the Hooke's law as

$$\begin{Bmatrix} \sigma_{xx}^{(k)} \\ \sigma_{xz}^{(k)} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11}^{(k)} & 0 \\ 0 & \bar{Q}_{55}^{(k)} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(k)} \\ \gamma_{xz}^{(k)} \end{Bmatrix} \qquad (9)$$

where $\bar{Q}_{11}$ and $\bar{Q}_{55}$ are the transformed axial and transverse ply properties, respectively [1].

The TBT equilibrium equations are derived within the framework of the principle of virtual work as

$$\delta U + \delta W = 0 \qquad (10)$$

where δ denotes the variational operator. The internal work, δU, is of the form

$$\delta U = \int_0^L \int_A \left(\sigma_{xx}^{(k)} \delta\varepsilon_{xx}^{(k)} + \sigma_{xz}^{(k)} \delta\gamma_{xz}^{(k)} \right) dAdx \qquad (11a)$$

or

$$\delta U = \int_0^L \left(N\delta u_{,x} + M\delta\theta_{,x} + S\delta(w_{,x} + \theta) \right) dx \quad (11b)$$

in which *N* and *M* are referred to stress and bending moment resultants, respectively, while S is the shear forces. The explicit form of *N*, *M*, and *S* can be expressed in terms of the displacement components defined in Eq. (7) as

$$\begin{Bmatrix} N \\ M \\ S \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & k_s^2 E \end{bmatrix} \begin{Bmatrix} u_{,x} \\ \theta_{,x} \\ w_{,x} + \theta \end{Bmatrix} \qquad (12)$$

where k_s^2 is the shear correction factor minimizes the errors arising from the assumption of constant transverse shear stress variations through the thickness of the laminate. The material parameters *A*, *B*, *D*, and *E* are given as

$$(A, B, D, E) = \int_{-h}^{h} (\bar{Q}_{11}, z\bar{Q}_{11}, z^2\bar{Q}_{11}, \bar{Q}_{55}) dz \qquad (13)$$

The virtual work done by external forces δW, is expressed in the form of

$$\delta W = \int_0^L p(x)\delta w dx \qquad (14)$$

where $p = p(x)$ is the external transverse distributed load.

The TBT equilibrium equations can be achieved by rearranging Eq. (12) with crossectional-integral and integration by parts as

$$N_{,x} = 0 \qquad (15a)$$

$$S_{,x} + p = 0 \qquad (15b)$$

$$M_{,x} - S = 0 \qquad (15c)$$

and boundary conditions:

$$\delta u = 0 \quad \text{or} \quad N = 0 \qquad (16a)$$

$$\delta\theta = 0 \quad \text{or} \quad M = 0 \qquad (16b)$$

$$\delta w = 0 \quad \text{or} \quad S = 0 \qquad (16c)$$

Substituting the kinematic variables of TBT and constitutive relations into Eq. (15), the equilibrium equations can be expressed as

$$\delta u: \ Au_{,xx} + B\theta_{,xx} = 0 \qquad (17a)$$

$$\delta\theta: \ Bu_{,xx} + D\theta_{,xx} - k_s^2 E(w_{,x} + \theta) = 0 \qquad (17b)$$

$$\delta w: \ k_s^2 E\left(w_{,xx} + \theta_{,x} \right) + p = 0 \qquad (17c)$$

The derivatives of TBT kinematic variables (u, θ, and w) in Eq. (17) can be written in their nonlocal PD form by employing Eq.(6).

4. NUMERICAL RESULTS

This section concerns the accuracy and robustness of the present approach PD-TBT for the bending behavior of multilayered composite structures. The effects of stacking sequences of material layers, span-to-thickness ratios, and boundary conditions are presented herein. The PD-TBT predictions for displacement and stress fields are verified with the reference solutions [11,25,26]. The shear correction factor is specified as $k_s^2 = 5/6$. The below material properties are used in the analyses in the form

$$E_1 / E_2 = 25,\ E_2 = E_3,$$
$$G_{12} = G_{13} = 0.5E_2,\ G_{23} = 0.2E_2, \qquad (18)$$
$$\nu_{12} = \nu_{13} = \nu_{23} = 0.25$$

Two different boundary conditions are considered with simply supported (S-S) and clamped (C-C) as shown in Fig. 3. The geometry of both beam is identical. The length and width of both beam are specified as $L = 1\text{m}$ and $b = 0.1\text{m}$, respectively.

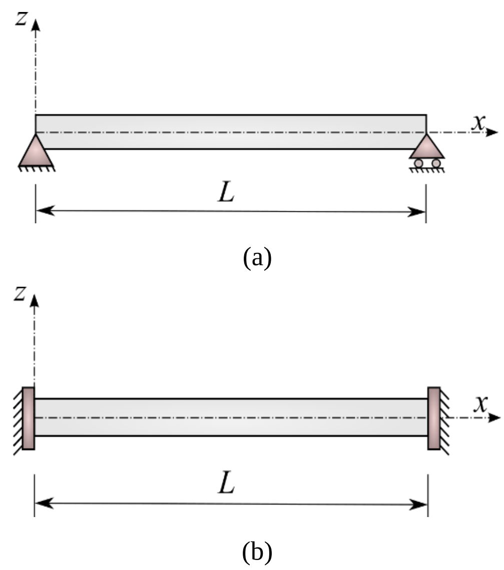

Figure 3. (a) Simply supported-simply supported (S-S) and (b) clamped-clamped (C-C) beams

The beams are subjected to uniformly and sinusoidal distributed transverse load, respectively, in the form

$$p = p_0(x) \qquad (19a)$$

and

$$p = p_0 sin(\pi x / L) \qquad (19b)$$

The simply supported (S) and clamped (C) boundary conditions can be enforced as explained in Eq. (16)

$$w = N_x = M_x = 0 \qquad (20a)$$

and

$$u = w = \theta = 0 \qquad (20b)$$

respectively. The deflection, axial, and transverse shear stress components can be expressed in their non-dimensional forms as

$$\bar{w} = \frac{10^2 E_2 b(2h)^3}{p_0 L^4} w(L/2, z) \qquad (21a)$$

$$\bar{\sigma}_{xx} = \frac{b(2h)^2}{p_0 L^2} \sigma_{xx}(L/2, z) \qquad (21b)$$

$$\bar{\sigma}_{xz} = \frac{b(2h)}{p_0 L} \sigma_{xz}(0, z) \qquad (21c)$$

The influence of each interaction in the family of PD point x depends on the weight function , $w(\xi)$, defined in Eq. (6) which can be chosen any non-dimensional value. In this study, the weight function is specified $w(\xi) = e^{-(2\xi/\delta)^2}$ for the representation of a more realistic interaction between the PD point x and its neighbors. Also, the grid spacings and horizon size play a major role on the efficient and accurate PD analyses. As previously elucidated in Ref [21,22], an optimum grid spacing and horizon size were obtained by performing a convergence study with various beam models. Hence, the PD solution domain for beams is discretized with a uniform grid spacing of $\Delta x = L/50$ and horizon size of $\delta = 4\Delta x$.

Problem: In this problem, laminated beams have symmetric and non-symmetric cross-sections. The symmetric beam has three layers with a stacking sequence of $[0^\circ/\theta^\circ/0^\circ]$ while the non-symmetric beam consists of two material layers $[0^\circ/\theta^\circ]$. The thicknesses of each material layer in the two and three-layered beams are specified as $2h^{(k)} = 2h/2$ and $2h/3$ m , respectively. Three different aspect ratios ($\rho = L/2h = 5,10$, and 50) and ply angles ($\theta^\circ = 0,45$, and 90) are used for modeling beams.

Table 1 presents the normalized deflection, axial and transverse shear stress values for the simply supported beam under uniformly distributed load.

The normalized mid-span deflections, $\bar{w}$, for $\theta = 90^\circ$ are compared with those of obtained by Khdeir and Reddy [25]. The normalized mid-span deflections ($\bar{w}$), axial ($\bar{\sigma}_{xx}$) and transverse shear ($\bar{\sigma}_{xz}$) stress values obtained from the PD-TBT for each fiber angle are compared with those of TBT solutions predicted by Karamanli [11]. As can be seen, the PD-TBT predictions correlate well with the reference solutions. It is worth noting that the transverse shear stresses are directly calculated from Hooke's law. The influence of fiber angles and aspect ratios as well as lamination plays an important role on the deflection and stress components. The deflection and transverse shear stress levels result in increases with increasing the fiber angle whereas axial stress concentrations decrease. It is clearly seen that the deflection levels increase with decreasing the thickness of beams. Also, three layered beam increases the structural strength, decreasing the deflection under loads.

Table 2 shows the influence of boundary conditions on the deflection levels and comparisons of the PD-TBT deflections against EBT and TBT [11]. In the table, the deflection levels for clamped symmetric and non-symmetric beams under uniformly distributed load are listed for three different aspect ratios. It is obvious that the differences in deflection levels decrease between EBT and TBT with decreasing the thickness of the beam. The clamped boundary conditions prevent the rotations at both ends. Therefore, the beams experience less deflections in comparison with those of simply supported beams (Table 1).

The load effects are crucial on the deflection behavior of laminated composites. Table 3 presents the deflection, axial and transverse shear stress predictions obtained from the PD-TBT, EBT, and TBT [26] for the simply supported beams under sinusoidally distributed transverse load. The results are acquired for three different span-to-thickness ratios. The load effects concentrate at the center of beams for sinusoidally distributed loads. Therefore, the deflections at the center of beams are higher comparing to the deflections captured under uniformly distributed loads. It is evident that the PD-TBT results are in good agreement with those of TBT results. The EBT theory fails to produce expected deflection levels when the thickness of the beam is increased.

Figures 4(a) and (b) illustrate the normalized through-thickness axial stress variations at the center of the beam and transverse shear stress variations at the left end of the beam for non-symmetric simply supported beam under uniformly distributed load whilst Figures 4(c) and (d) depicts these components for the symmetric beam.

Table 1. Comparison of the PD-TBT predictions for normalized deflection and stress components with TBT solutions [11,25] for the simply supported beam under uniformly distributed transverse load

Lamination	ρ	θ	PD-TBT $\bar{w}$	PD-TBT $\bar{\sigma}_{xx}$	PD-TBT $\bar{\sigma}_{xz}$	TBT $\bar{w}$	TBT $\bar{\sigma}_{xx}$	TBT $\bar{\sigma}_{xz}$
$[0^{\circ}/\theta^{\circ}]$	5	0	1.8234	0.7500	0.6000	1.8234	0.7500	0.6000
		45	2.6756	0.5538	0.7059	2.6757	0.5538	0.7059
		90	5.0358	0.2336	0.8571	5.0359	0.2336	0.8571
	10	0	0.9234	0.7500	0.6000	0.9234	0.7500	0.6000
		45	1.6168	0.5538	0.7059	1.6169	0.5538	0.7059
		90	3.7502	0.2336	0.8571	3.7502	0.2336	0.8571
	50	0	0.6354	0.7500	0.6000	0.6354	0.7500	0.6000
		45	1.2781	0.5538	0.7059	1.2780	0.5538	0.7059
		90	3.3387	0.2336	0.8571	3.3387	0.2336	0.8571
$[0^{\circ}/\theta^{\circ}/0^{\circ}]$	5	0	1.8234	0.7500	0.6000	1.8234	0.7500	0.6000
		45	1.9737	0.7704	0.4667	1.9737	0.7704	0.4667
		90	2.1464	0.7776	0.3000	2.1464	0.7776	0.3000
	10	0	0.9234	0.7500	0.6000	0.9234	0.7500	0.6000
		45	0.9737	0.7704	0.4667	0.9737	0.7704	0.4667
		90	1.0214	0.7776	0.3000	1.0214	0.7776	0.3000
	50	0	0.6354	0.7500	0.6000	0.6354	0.7500	0.6000
		45	0.6537	0.7704	0.4667	0.6537	0.7704	0.4667
		90	0.6614	0.7776	0.3000	0.6614	0.7776	0.3000

Table 2. Deflection, $\bar{w}(L/2,0)$, of symmetric and non-symmetric clamped beams under uniformly distributed load

Lamination	Theory	ρ = 5	10	50
$[0^{\circ}/90^{\circ}]$	PD-TBT	2.3786	1.0929	0.6815
	TBT	2.3786	1.0929	0.6815
	EBT	0.6643	0.6643	0.6643
$[0^{\circ}/90^{\circ}/0^{\circ}]$	PD-TBT	1.6293	0.5043	0.1443
	TBT	1.6293	0.5043	0.1443
	EBT	0.1293	0.1293	0.1293

The axial stresses concentrate the bottom surface of the beam. The axial stress levels become maximal in the non-symmetric beam with 90º fiber angle in the bottom layer. Also, a jump occurs along the material layer interfaces. This behavior can be successfully captured by the PD-TBT. The transverse shear stresses exhibit constant variation in each layer when they are directly calculated from Hooke’s law. This variation of transverse shear stresses violates the traction free boundary conditions along the top and bottom layers.

Table 3. Comparison of the PD-TBT predictions for normalized deflection and stress components with EBT and TBT solutions [26] for the simply supported beam under sinusoidally distributed load

ρ	Theory	$[0^\circ/90^\circ]$			$[0^\circ/90^\circ/0^\circ]$		
		$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{xz}$	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{xz}$
	PD-TBT	4.0149	0.1894	0.5457	1.7268	0.6303	0.1910
5	TBT	4.0149	0.1894	0.5457	1.7268	0.6303	0.1910
	EBT	2.6254	0.1894	-	0.5109	0.6303	-
	PD-TBT	2.9728	0.1894	0.5457	0.8149	0.6303	0.1910
10	TBT	2.9728	0.1894	0.5457	0.8149	0.6303	0.1910
	EBT	2.6254	0.1894	-	0.5109	0.6303	-
	PD-TBT	2.6393	0.1894	0.5457	0.5231	0.6303	0.1910
50	TBT	2.6393	0.1894	0.5457	0.5231	0.6303	0.1910
	EBT	2.6254	0.1894	-	0.5109	0.6303	-

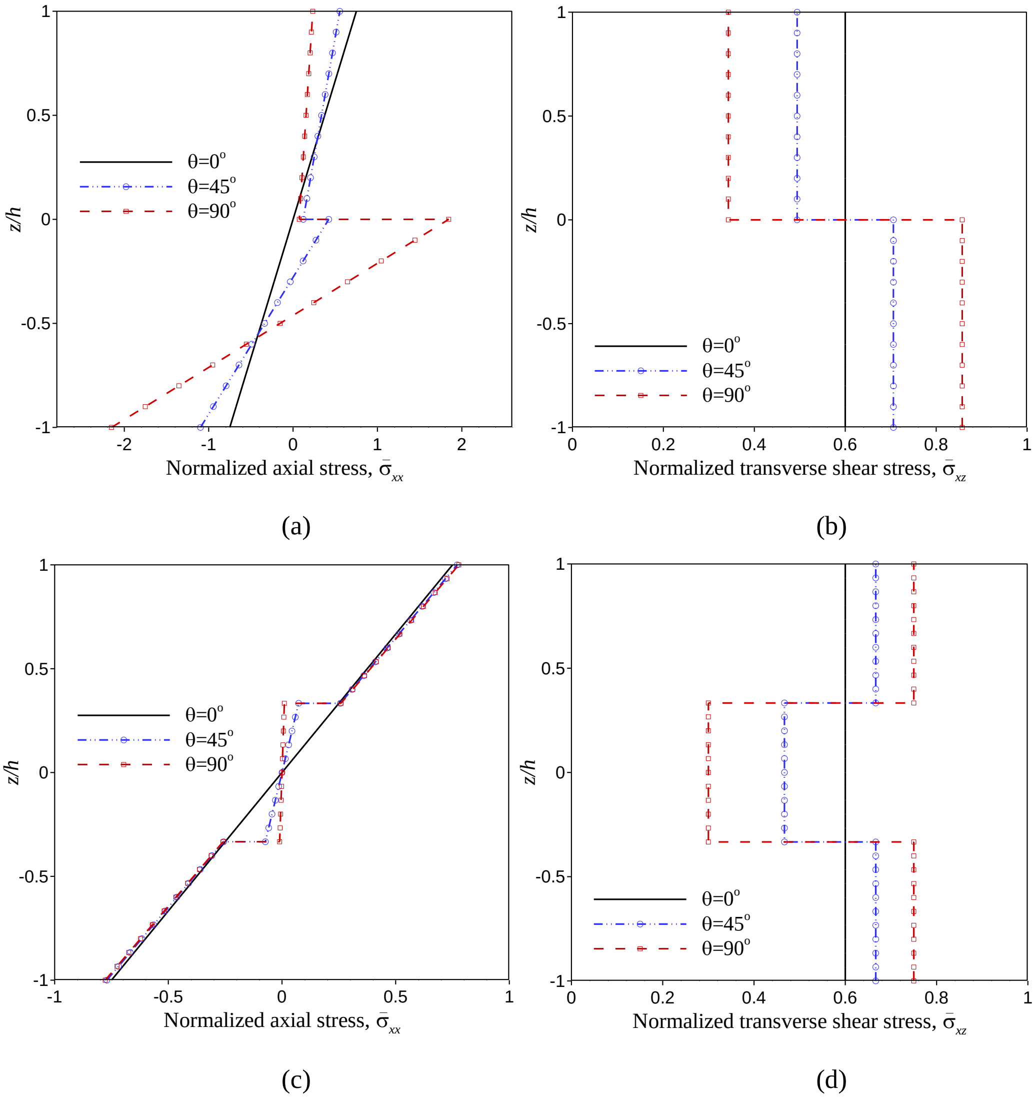

Figure 4. Variations of normalized (a) axial and (b) transverse shear stress for simply supported non-symmetric beam, (c) axial and (d) transverse shear stress for simply supported symmetric beam under uniformly distributed load ($\rho = 10$)

The accurate determination of transverse shear components is of vital importance for the failure analysis. In order to eliminate the discontinuities in the transverse shear stresses along the interface and satisfy the traction free boundary conditions along the top and bottom surfaces, the stress equilibrium equation is employed as given

$$\sigma_{xz} = -\int_{-h}^{z} \sigma_{xx,x} dz \,. \tag{22}$$

Here, the spatial derivative of the axial stress can be determined by means of PDLSM in the nonlocal form. Thus, the PD form of the axial stress component is achieved through Eq. (6).

Figure 5 shows the transverse shear stress variations through the thickness of the simply supported two- and three-layered beams under the sinusoidally distributed load. In order to highlight the importance of the way of continuous transverse shear stress variations, they are calculated by using both Hooke's law (HL) and stress equilibrium equations (EQ). The through-thickness transverse shear stress variations are obtained at the left end of the beam.

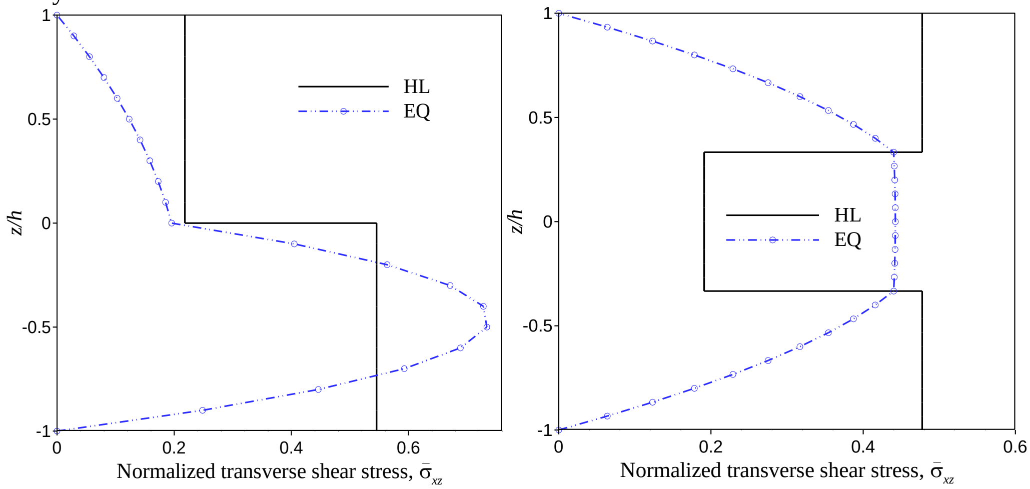

Figure 5. Variations of normalized transverse shear stress for simply supported (a) non-symmetric and (b) symmetric beams under sinusoidally distributed load ($\rho = 10$)

As can be seen, the transverse shear stress variation is constant through the thickness of the beam, violating the zero traction boundary conditions along the top and bottom surfaces of the beam in the case of Hooke's law. It is evident that the through-thickness transverse stress variation directly obtained from Hooke's law deviates from those obtained stress equilibrium equations. The transverse stresses determined from the stress equilibrium equations experience parabolic variations along the cross-section and disappear along the free surfaces as anticipated.

5. CONCLUSIONS

In this study, the PDLSM and TBT were coupled for the stress and deformation analysis of laminated composites considering different span-to-thickness ratios, loading and boundary conditions as well as laminations. The PDLSM determines any order derivatives of a function in their nonlocal forms with high accuracy and is highly suitable for the solution of TBT equilibrium equations. The accuracy of the PD-TBT was demonstrated by comparing the stress and displacement predictions against different reference solutions. Also, the robustness of the PDLSM was illustrated on the transverse shear stress calculations from stress equilibrium equations. It was obvious that the transverse shear stress variations computed directly from Hooke's law were inferior since they had a constant variation in each layer and were not able to satisfy zero traction boundary conditions along the top and bottom layers of the beam.

The influence of boundary/loading conditions, lamination, and span-to-thickness ratioes were considered. It was found that increasing the fiber angles led to higher mid-span deflection and transverse shear stress levels, unlike the axial stress levels. The laminated beams experienced higher deflection levels with decreasing the total thickness. It was observed that the structural strength was improved with the use of three layered beam. Although the EBT predictions deviated from the TBT results for thick beams, the EBT and TBT theories produced similar deflection levels with decreasing the thickness of the beam. The clamped beams underwent less deflection levels comparing to those of simply supported beams since it fixed rotations at the ends of beams.

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VITAE

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